This note estimates the effect of scintillator thickness on track measurements. The approach used to describe muon tracks and the relevant nomenclature are presented in ref. [1]. Track parameters are determined from entrance and exit coordinates \((\phi_{en}, z_{en} \text{ and } \phi_{ex}, z_{ex} \text{ respectively})\) measured where the muon intersects the cylindrical tracking layers of the detector (described in ref. [2]).

Assuming a given muon intersection results in exactly one hit scintillator strip from each layer (a “triplet” labeled by coordinates \(\phi_{+}, \phi_{-}, \phi_{0}\)), the least-square estimate for the location of each intersection point \(\phi, z\) is given by:

\[
\begin{align*}
\phi &= \frac{\phi_{+} + \phi_{-} + (1 + \lambda^2) \phi_{0}}{3 + \lambda^2} \\
z &= \frac{\phi_{-} - \phi_{+}}{2\lambda}
\end{align*}
\] (1)

where \(\lambda \equiv \tan \theta_s\); \(\theta_s\) is the stereo angle of the tracking system, 30° in the present case.

The arrangement of scintillator strips is indicated in Fig. 1. In this note, it is assumed that the axial strips are mounted on a cylindrical surface between the two layers of helical strips. The radius of the centers of the axial strips is taken to be the unit-of-distance scale. The thickness of each tracking layer (in units of the radius of the axial layer) is denoted by \(t\).

Fig. 2 indicates the distance of closest approach \(b\) and the projected chord \(c\) of the track as it crosses the cylindrical detector. These quantities are related to the entrance and exit azimuthal coordinates by:

\[
\begin{align*}
b &= \cos \left( \frac{\phi_{en} - \phi_{ex}}{2} \right) \\
c &= 2 \sin \left( \frac{\phi_{en} - \phi_{ex}}{2} \right) = 2\sqrt{1 - b^2}
\end{align*}
\] (4)
Figure 1: An “unwrapped” representation of the detector cylinder indicating the scintillator strips hit by a muon at the coordinate $\phi, z$. The strips hit are identified by coordinates $(\phi_+, \phi_-, \phi_0)$ on the two stereo layers and single axial layer, respectively. It is assumed that the innermost tracking layer is the stereo layer represented by $\phi_-$, the middle layer is the axial layer ($\phi_0$) and the outer layer is the other stereo layer ($\phi_+$).
Figure 2: Detector geometry indicating muon trajectory, direction angles and hit locations. a) indicates a vertical plane which includes the muon track; b) horizontal plane indicating the three tracking layers. The subscripts “en” and “ex” indicate the entrance and exit locations of the track at the detector surface. The thickness corrections given by $\Delta z, \Delta \phi$ are indicated. The radius of the central tracking layer, the circle in b), is taken to have unit radius.
The reconstructed track azimuth and zenith angles are found from:

\[ \Phi = \frac{\phi_{en} + \phi_{ex}}{2} + \frac{\pi}{2} \quad (5) \]
\[ \cot \Theta = \frac{z_{en} - z_{ex}}{c} \quad (6) \]

The coordinates \( X, Y, Z \) of the point of closest approach of the muon track to the detector axis are related to muon hit coordinates and track direction parameters by:

\[ X = b \sin \Phi \]
\[ Y = -b \cos \Phi \]
\[ Z = \frac{z_{en} + z_{ex}}{2} \quad (7) \]

Fig. 2 also indicates corrections \( \Delta z, \Delta \phi \) to the intersections of a track with the two stereo tracking layers arising from the non-zero thickness \( t \) of the scintillator strips. By symmetry, these corrections affect the entrance and exit coordinates by the same amounts, with the sign of the appropriate correction as shown in the figure. From elementary geometry, the thickness corrections are found to be:

\[ \Delta z = 2t \frac{c}{\cot \Theta} \quad (8) \]
\[ \Delta \phi = 2t \frac{b}{c} \quad (9) \]

where the nominal track parameters \( b, c \) and \( \cot \Theta \) are determined from the scintillator strip information assuming zero thickness. These parameters do depend on the scintillator strip thickness; corrections to the zero-thickness values are estimated next. The remaining two independent track parameters \( \Phi, Z \) do not depend on the scintillator thickness.

Keeping correction terms to first-order in \( t \), the corrections to the zero-thickness approximations for the track parameters \( b, \cot \Theta \) are*:

\[ \Delta b = -t \left( \frac{2\lambda}{3 + \lambda^2} \right) \cot \Theta \quad (10) \]
\[ \Delta \cot \Theta = t \left( \frac{b}{1 - b^2} \right) \left[ \frac{1}{\lambda} - \left( \frac{2\lambda}{3 + \lambda^2} \right) \cot^2 \Theta \right] \quad (11) \]

*Eqns. 10 and 11 assume triplets of hit strips contribute to the estimation of \( \phi_{en} \) and \( \phi_{ex} \). If only the stereo layers are used to compute these coordinates, then the term \( 2\lambda/(3 + \lambda^2) \) used in these expressions should be replaced by \( \lambda/2 \).
The suggested procedure for applying the thickness corrections is:

1. Compute the entrance and exit coordinates \((\phi_{en}, z_{en}, \phi_{ex}, z_{ex})\) from the coordinates of hit strips using Eqs. 1, 2 which assume the tracking layers have zero thickness. Compute the corresponding track parameters \((b, \cot \Theta, \Phi, Z)\) using these coordinates.

2. Update the track parameters by the replacements \((b + \Delta b, \cot \Theta + \Delta \cot \Theta, \Phi, Z)\) computed from Eqs. 10, 11.

3. The goodness-of-fit measure \(\chi_{\text{min}}^2\) which can be used to test the consistency of the three hits in a triplet (see ref. [1]) should be modified to account for thickness effects. The appropriate form is:

\[
\chi_{\text{min}}^2 = \frac{24}{3 + \lambda^2} \left[ \frac{(\phi_+ + \phi_-) / 2 \pm \lambda \Delta z - \phi_0}{w} \right]^2
\]  

(12)

where the choice of sign on the \(\lambda \Delta z\) term (computed from Eq. 8) depends on whether \(\chi^2\) is being computed for the entrance triplet of hit strips or the exit triplet.

References
